

Trig. Identities Worksheet #2

Name KEY-FOCHTMAN

Date 11-17-15

Prove each identity.

$$\begin{aligned}
 1. \quad \frac{\cos \alpha}{1 + \sin \alpha} + \frac{1 + \sin \alpha}{\cos \alpha} &= 2 \sec \alpha \\
 \frac{\cos^2 \alpha + 1 + 2 \sin \alpha + \sin^2 \alpha}{(1 + \sin \alpha) \cos \alpha} & \\
 = \frac{1 + 1 + 2 \sin \alpha}{(1 + \sin \alpha) \cos \alpha} &= \frac{2 + 2 \sin \alpha}{(1 + \sin \alpha) \cos \alpha} = \frac{2(1 + \sin \alpha)}{(1 + \sin \alpha) \cos \alpha} \\
 &= \frac{2}{\cos \alpha} = \boxed{2 \sec \alpha}
 \end{aligned}$$

$$\begin{aligned}
 2. \quad \frac{\cos x}{1 - \sin x} - \frac{\cos x}{1 + \sin x} &= 2 \tan x \\
 \frac{(1 + \sin x) \cos x - \cos x (1 - \sin x)}{(1 - \sin x)(1 + \sin x)} &= \frac{\cos x}{1 - \sin^2 x} \\
 = \frac{\cos x + \cos x \sin x - \cos x + \cos x \sin x}{1 - \sin^2 x} &= \frac{2 \cos x \sin x}{\cos^2 x} = \frac{2 \sin x}{\cos x} = \boxed{2 \tan x} \checkmark
 \end{aligned}$$

$$\begin{aligned}
 3. \quad \cos^2 x &= \frac{\csc x \cos x}{\tan x + \cot x} \\
 &= \frac{\frac{1}{\sin \theta} \cdot \cos \theta}{\frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta}} \\
 &= \frac{\frac{\cos \theta}{\sin \theta}}{\frac{\sin^2 \theta + \cos^2 \theta}{\sin \theta \cos \theta}} = \frac{\cos \theta}{\sin \theta} \cdot \frac{\sin \theta \cos \theta}{1} \\
 &= \frac{\cos \theta}{\sin \theta} \cdot \frac{\sin \theta \cos \theta}{\sin \theta \cos \theta} = \boxed{\cos^2 \theta}
 \end{aligned}$$

$$\begin{aligned}
 4. \quad \frac{\sin^4 x - \cos^4 x}{\sin^2 x - \cos^2 x} &= 1 \\
 \frac{(\sin^2 x + \cos^2 x)(\sin^2 x - \cos^2 x)}{\sin^2 x - \cos^2 x} &= \frac{1 \cdot (\sin^2 x - \cos^2 x)}{\sin^2 x - \cos^2 x} = \boxed{1} \checkmark
 \end{aligned}$$

$$\begin{aligned}
 5. \quad \frac{\tan^2 x}{\tan^2 x + 1} &= \sin^2 x \\
 \frac{\frac{\sin^2 x}{\cos^2 x}}{\frac{\sin^2 x}{\cos^2 x} + \frac{1 \cdot \cos^2 x}{\cos^2 x}} &= \frac{\frac{\sin^2 x}{\cos^2 x}}{\frac{\sin^2 x + \cos^2 x}{\cos^2 x}} = \frac{\sin^2 x}{\sin^2 x + \cos^2 x} \\
 &= \frac{\sin^2 x}{1} = \boxed{\sin^2 x}
 \end{aligned}$$

$$\begin{aligned}
 6. \quad 1 - 2 \cos^2 x &= \frac{\tan^2 x - 1}{\tan^2 x + 1} \quad | -\cos^2 x = \sin^2 x \\
 &= \frac{\frac{\sin^2 x}{\cos^2 x} - \frac{1 \cdot \cos^2 x}{1 \cdot \cos^2 x}}{\frac{\sin^2 x}{\cos^2 x} + \frac{1 \cdot \cos^2 x}{1 \cdot \cos^2 x}} \\
 &= \frac{\frac{\sin^2 x - \cos^2 x}{\cos^2 x}}{\frac{\sin^2 x + \cos^2 x}{\cos^2 x}} = \frac{\sin^2 x - \cos^2 x}{\sin^2 x + \cos^2 x} \\
 &= \frac{\sin^2 x - \cos^2 x}{1} = \frac{\sin^2 x - \cos^2 x}{1} \\
 &= \frac{\sin^2 x + \cos^2 x}{\cos^2 x} = \frac{1 - \cos^2 x - \cos^2 x}{\cos^2 x} = \frac{1 - 2 \cos^2 x}{\cos^2 x} \checkmark
 \end{aligned}$$

$$\begin{aligned}
 7. \quad \tan^2 \theta &= \csc^2 \theta \tan^2 \theta - 1 \\
 &= \frac{1}{\sin^2 \theta} \cdot \frac{\sin^2 \theta}{\cos^2 \theta} - 1 \\
 &= \frac{1}{\cos^2 \theta} - 1 \\
 &= \sec^2 \theta - 1 \\
 &= \tan^2 \theta \quad \checkmark
 \end{aligned}$$

$$\begin{aligned}
 8. \quad \sec x + \tan x &= \frac{\cos x}{1 - \sin x} \cdot \frac{1 + \sin x}{1 + \sin x} \\
 &= \frac{\cos x (1 + \sin x)}{(1 - \sin x)(1 + \sin x)} \\
 &= \frac{\cos x + \sin x \cos x}{1 - \sin^2 x} \\
 &= \frac{\cos x + \sin x \cos x}{\cos^2 x} \\
 &= \frac{\cos x}{\cos^2 x} + \frac{\sin x \cos x}{\cos^2 x} = \frac{1}{\cos x} + \frac{\sin x}{\cos x} = \sec x + \tan x \quad \checkmark \\
 \sin^2 x &= 1 - \cos^2 x
 \end{aligned}$$

$$\begin{aligned}
 9. \quad \frac{\csc \beta}{\sin \beta} - \frac{\cot \beta}{\tan \beta} &= 1 \\
 &= \frac{1}{\sin \beta} - \frac{\cos \beta}{\sin \beta} \\
 &= \frac{1}{\sin \beta} \cdot \frac{1}{\sin \beta} - \frac{\cos \beta \cdot \cos \beta}{\sin \beta \cdot \sin \beta} \\
 &= \frac{1 - \cos^2 \beta}{\sin^2 \beta} = \frac{\sin^2 \beta}{\sin^2 \beta} = 1 \quad \checkmark
 \end{aligned}$$

$$\begin{aligned}
 10. \quad \sin^4 x - \cos^4 x &= 1 - 2\cos^2 x \\
 &= (\sin^2 x + \cos^2 x)(\sin^2 x - \cos^2 x) \\
 &= 1(\sin^2 x - \cos^2 x) \\
 &= 1 - \cos^2 x - \cos^2 x \\
 &= 1 - 2\cos^2 x \quad \checkmark
 \end{aligned}$$

$$\begin{aligned}
 11. \quad (\sin x - \cos x)^2 + (\sin x + \cos x)^2 &= 2 \\
 &= \sin^2 x - 2\sin x \cos x + \cos^2 x + \sin^2 x + 2\sin x \cos x + \cos^2 x \\
 &= 2\sin^2 x + 2\cos^2 x \\
 &= 2(\sin^2 x + \cos^2 x) \\
 &= 2 \cdot 1 = 2 \quad \checkmark
 \end{aligned}$$

$$\begin{aligned}
 12. \quad \tan^2 x + 1 + \tan x \sec x &= \frac{1 + \sin x}{\cos^2 x} \\
 &= \frac{\sin^2 x}{\cos^2 x} + \frac{1 + \sin x}{1} \cdot \frac{1}{\cos x} \\
 &= \frac{\sin^2 x}{\cos^2 x} + \frac{\cos^2 x}{\cos^2 x} + \frac{\sin x}{\cos^2 x} \\
 &= \frac{\sin^2 x + \cos^2 x + \sin x}{\cos^2 x} \\
 &= \frac{1 + \sin x}{\cos^2 x} \quad \checkmark
 \end{aligned}$$