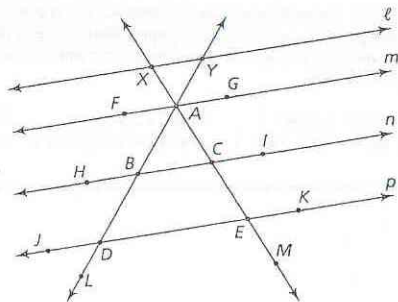


Topic 6: Parallel and Perpendicular

for use after *The Shapes of Algebra* **Investigation 2**

In the diagram below, lines ℓ , m , n , and p are parallel lines. The other two lines are transversals. Angles ACI and CEK are in corresponding positions at the vertices C and E . Each is in the “top-right” or “north-east” position at their vertices. Because of their corresponding positions, they are called **corresponding angles**. Corresponding angles are congruent to each other if they are formed by a transversal intersecting parallel lines. Angles BAC and XAY are **vertical angles**. Vertical angles are always congruent to each other.



When parallel lines are cut by non-parallel transversals, similar triangles are formed. In this figure, triangle ABC is similar to triangle ADE . Corresponding sides of these similar triangles form equal ratios. For example:

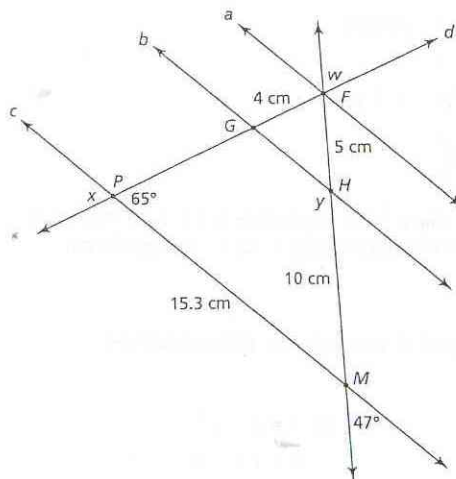
$$\frac{\text{length of } AC}{\text{length of } AE} = \frac{\text{length of } AB}{\text{length of } AD}$$

Exercises

For Exercises 1–6, use the diagram above.

- List five other pairs of vertical angles in the diagram.
 - List five other pairs of corresponding angles in the diagram.
- What other segments form equal ratios? Explain.
- What angles are congruent to $\angle CEK$? Explain.
- What angles are congruent to $\angle BDJ$? Explain.
- Why are $\angle EDL$ and $\angle BDJ$ congruent?
- What other triangle is similar to triangle ABC ? Explain.

For Exercises 7–13, use the diagram below. Lines a , b , and c are parallel.



- What are the measures of $\angle PMH$ and $\angle x$?
- What are the measures of $\angle FGH$ and $\angle GHF$?
- What is the measure of $\angle PGH$?
- What is the measure of $\angle y$?
- What is the length of \overline{GP} ?
- What is the measure of $\angle GFH$ and $\angle w$?
- What is the length of \overline{GH} ?

Remember that lines are parallel if their slopes are equal and lines are perpendicular if their slopes are negative reciprocals of each other.

Sample Are the lines $y = 2x + 8$ and $3 = 2x - y$ parallel?

Rewrite the second equation.

$$\begin{aligned} 3 &= 2x - y \\ y + 3 &= 2x - y + y \\ y + 3 &= 2x \\ y &= 2x - 3 \end{aligned}$$

The slope of this line is 2, which is also the slope of the first line. The slopes are equal, so the lines are parallel.

Sample Are the lines $y = 2x + 8$ and $7 = \frac{1}{3}x - y$ perpendicular?

Rewrite the second equation.

$$7 = \frac{1}{3}x - y$$

$$y + 7 = \frac{1}{3}x - y + y$$

$$y + 7 = \frac{1}{3}x$$

$$y = \frac{1}{3}x - 7$$

The slope of this line is $\frac{1}{3}$. The slope of the first line is 2. The slopes are not negative reciprocals of each other, so the lines are not perpendicular.

Determine whether each pair of lines is parallel, perpendicular, or neither.

14. $y = 5x - 7$

$$y + 5x = 12$$

15. $y = x - 0.5$

$$y + x = 0.25$$

16. $y = \frac{1}{2}x - \frac{3}{4}$

$$y - \frac{1}{2}x = \frac{5}{6}$$

17. $2y = 6x - 72$

$$y - 3x = 15$$

18. $y + x = 12$

$$y - x = 12$$

19. $5x - y = 12$

$$5y + x = 35$$

Graphing \parallel and \perp lines

Students will graph and label equations of the form $y = mx + b$ to learn about the slopes of parallel and perpendicular lines. To study parallel lines, students will graph and label the pairs of lines given below on separate graphs. After graphing the equations, students will describe in their own words what lines with the same slope look like geometrically.

1 slope = 4, y-int = 1

3 slope = -2, y-int = -3

5 slope = 1, y-int = 0

2 slope = 4, y-int = 3

4 slope = -2, y-int = 1

6 slope = 1, y-int = -2

To study perpendicular lines, students will graph and label the pairs of lines given below on separate graphs. After graphing the equations, students will describe in their own words what lines with the same slope look like geometrically.

7 slope = 2, y-int = 2

9 slope = 3, y-int = 4

11 slope = 1, y-int = 0

8 slope = -1/2, y-int = 3

10 slope = -1/3, y-int = 2

12 slope = -1, y-int = 0